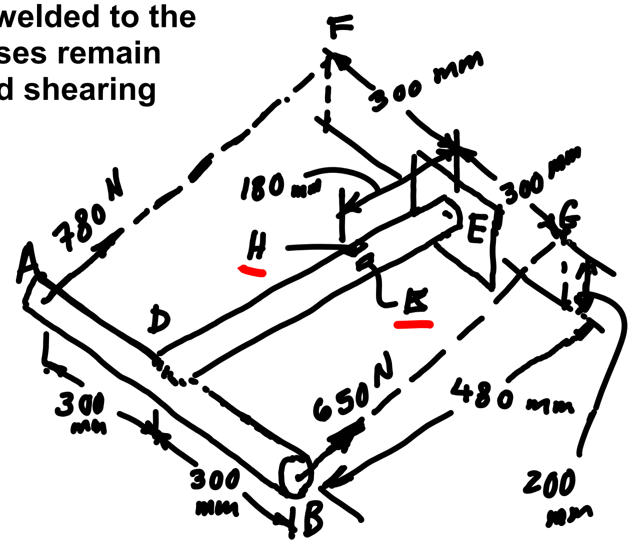
Note Title

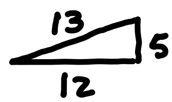
4-13 Combined Loading

Most often components are under combinations of loadings. We consider an example here.

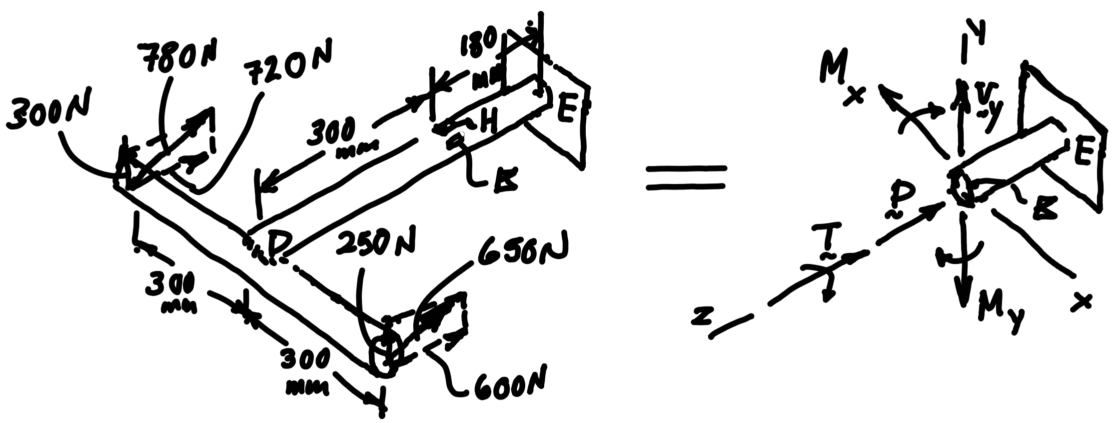
Example: Two forces are applied to rod AB, which is welded to the 50-mm diameter cylinder DE. Assuming that all stresses remain below the proportional limit, determine the normal and shearing stresses (a) at point H, and (b) at point K.



Slope of AF and BG :



Distance DH = 480 - 180 = 300 mm



Load Analysis

Force-couple in section containing H & K :

ΣF_x : $V_x = 0$

ΣF_y : $V_y = 300 + 250 = 550$ N ("+y" direction)

ΣF_z : $P = 720 + 600 = 1320$ N ("-z" direction : Compression)

ΣM_x : $M_x = (300 + 250)(0.300\ m) = 165$ N.m ("-x" direction)

ΣM_y : $M_y = (720 - 600)(0.300\ m) = 36$ N.m ("-y" direction)

ΣM_z : $T = (300 - 250)(0.300\ m) = 15$ N.m ("-z" direction)

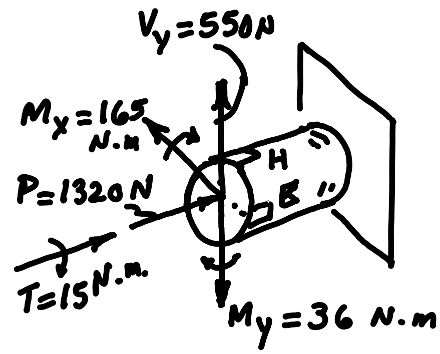
Geometric Properties

$c = 25\ mm = 0.025\ m$

$$A = \pi c^2 = \pi (0.025)^2 = 1.963 \times 10^{-3}\ m^2$$

$$I = \frac{\pi c^4}{4} = \frac{\pi}{4}(0.025)^4 = 306.8 \times 10^{-9}\ m^4$$

$$J = 2I = 613.6 \times 10^{-9}\ m^4$$

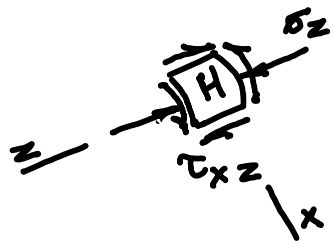


Stress State at Point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x c}{I} = \frac{-1320}{1.963 \times 10^{-3}} - \frac{165(.025)}{306.8 \times 10^{-9}}$$

$$= -14.12\ MPa = \sigma_z$$

$$\tau_H = \frac{Tc}{J} + 0 = \frac{15(.025)}{613.6 \times 10^{-9}} = .611\ MPa = \tau_{xz}$$



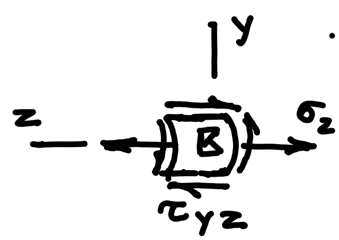
Stress State at Point K

$$\sigma_K = -\frac{P}{A} + \frac{M_y c}{I} = \frac{-1320}{1.963 \times 10^{-3}} + \frac{36(.025)}{306.8 \times 10^{-9}}$$

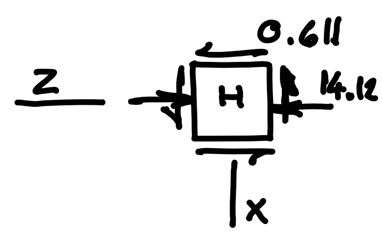
$$= 2.26\ MPa = \sigma_z$$

$$\tau_K = \frac{Tc}{J} - \frac{4V}{3A} = \frac{15(.025)}{613.6 \times 10^{-9}} - \frac{4(550)}{3(1.963 \times 10^{-3})}$$

$$= .237\ MPa = \tau_{yz}$$



Maximum Normal & Shear Stresses at H



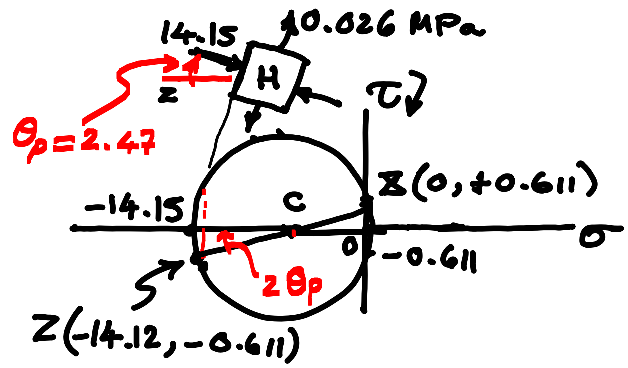
To avoid making a mistake in determining the sense of rotation, we can draw the Mohr's circle.

$z(-14.12, -.611)$

$x(0, +.611)$

$C(-7.06, 0)$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{.611}{14.12 - 7.06} = 2.47°$$



$$R = \overline{CX} = \sqrt{(-7.06-0)^2+(0-.611)^2} = 7.086 \text{ MPa}$$

$$\sigma_1 = R - \overline{OC} = 7.086 - 7.06 = .026 \text{ MPa}$$

$$\sigma_2 = -\{R + \overline{OC}\} = -(7.068+7.06) = -14.15 \text{ MPa}$$

The same results could be obtained analytically except that we should be careful about the sense of rotation:

$$\sigma_{1,2} = \frac{-14.12}{2} \pm \sqrt{\left(\frac{14.12}{2}\right)^2 + 0.611^2} = +0.026 \text{ MPa}, -14.15 \text{ MPa}$$

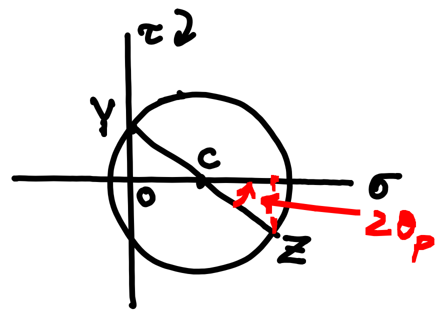
$$\tan 2\theta_p = \frac{2(0.611)}{-14.12} \Rightarrow \theta_p = -2.47^\circ = 2.47 \circlearrowright$$

Note that using the right-hand rule a positive rotation about the y-axis would be counter-clockwise. Hence, a negative rotation is clockwise.

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(0.026 + 14.15) = 7.086 \text{ MPa}$$

Maximum Normal & Shear Stresses at Point K

Similarly, for point K the Mohr's circle is as shown.



$$\left.\begin{array}{l} Z(+2.26, -0.237) \\ Y(0, +0.237) \end{array}\right\} C(1.13, 0)$$

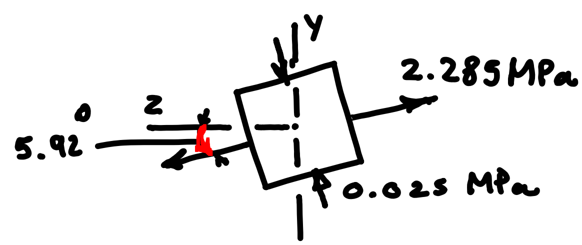
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{0.237}{2.26-1.13}, \quad \theta_p = +5.92^\circ = 5.92^\circ \circlearrowleft$$

σ_1, σ_2, τ_{max} are found to be

$$\sigma_1 = 2.285 \text{ MPa}$$

$$\sigma_2 = 0.025 \text{ MPa}$$

$$\tau_{max} = 1.13 \text{ MPa}$$



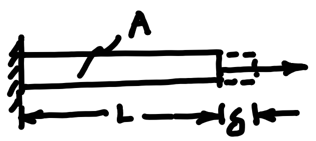
4.14 Spring Rates

A spring is a mechanical element that deflects under a force, or conversely, exerts a force when deflected.

Flexibility of a component is modeled by using the elastic properties of springs. Some examples of springs are *tension*, *compression*, and *torsion* springs.

Spring Type | **Deflection** | **Spring Rate**

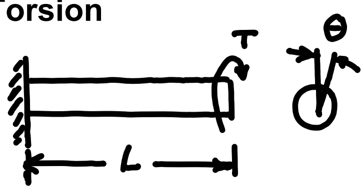
1. Tension or Compression



$$\delta = \frac{PL}{EA}$$

$$K = \frac{F}{x} = \frac{P}{\delta} = \frac{AE}{L}$$

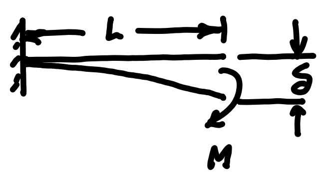
2. Torsion



$$\theta = \frac{TL}{GK}$$

$$K = \frac{T}{\theta} = \frac{KG}{L}$$

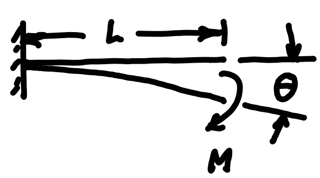
3. Bending (Linear Deflection)



$$\delta = \frac{ML^2}{2EI}$$

$$K = \frac{M}{\delta} = \frac{2EI}{L^2}$$

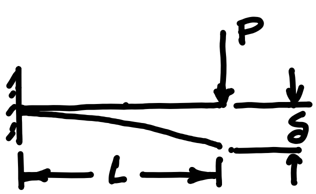
4. Bending (Angular Deflection)



$$\theta = \frac{ML}{EI}$$

$$K = \frac{M}{\theta} = \frac{EI}{L}$$

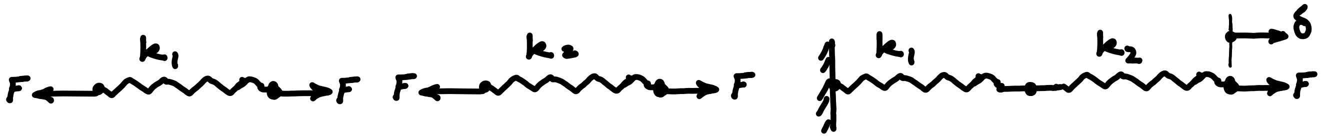
5. Cantilever Beam Loaded at End



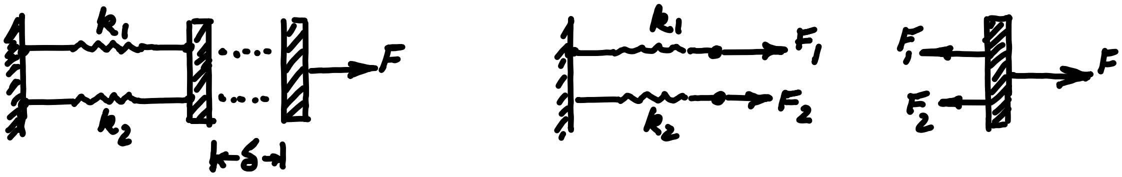
$$\delta = \frac{PL^3}{3EI}$$

In above expression δ is the linear and θ is the angular deflection.

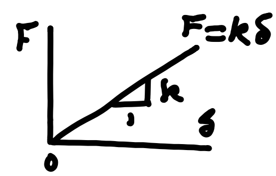
Springs are said to be attached in *series* if they are under the same force:



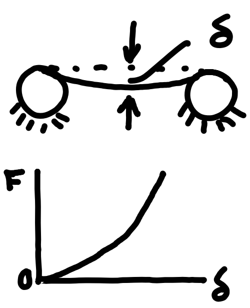
Springs are said to be attached in *parallel* if they undergo the same deflection:



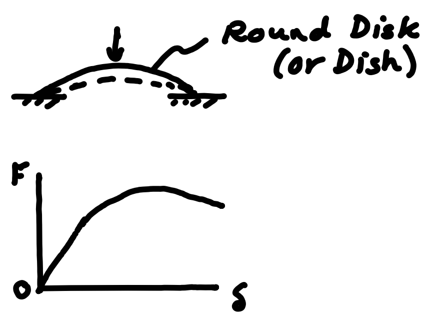
In above examples, springs are all linear, i.e., loads are within the proportional limit and F = kδ.



Here are some examples of "*non-linear*" springs:



Distance between supports decreases. A larger force is required to deflect a short beam than a long one.



The force needed to flatten the disk increases at first and then decreases as the disk approaches a flat configuration.

For non-linear springs, F = F(δ) and spring rate: is k = dF/dδ = F'(δ).

4.15 Stress Concentration

Most of the equations we have considered so far were derived assuming that no irregularities occurred in the region considered. However, in real world irregularities do occur, e.g.,

*** stepped shafts to allow for bearings**

*** bolts with threads**

*** holes, notches, grooves, and key slots in shafts/plates**

*** surface finish or irregularities**

The above irregularities (or discontinuities) are called "*stress raisers*" and the regions in which they occur are called areas of "*stress concentration*."

Stress concentration became an important topic of research in the 1930's & 1940's when machines designed on the basis of average and uniform stress distribution failed catastrophically even though designers adhered to the specific allowable stresses. It was later discovered that stress raisers, such as rivet holes (in the case of large navy ships) and approval stamps on shafts (in the case of aircrafts) were the source of catastrophic failures.

A widely used method for finding the stress state at a point, until 1980's was an experimental method called photoelasticity. Photoelasticity is based on the double refraction or birefingence property of transparent polymers. Doubly refracting, or birefringent polymers can split, or separate a single light ray into two orthogonally polarized rays moving at different speeds. Photoelastic birefringence is caused by the elastic deformation of a polymer. When a polymer in a solid glass state below T_g (the glass transition temperature, T_g, is the temperature at which the molecular chains are able to slide past each other when a force is applied) is stressed, the photoelastic birefringence is generated and the magnitude of the birefringence is in proportion to the applied stress. A significant characteristic of the birefringence is that it is eliminated by removing the stress.

Employing photoelasticity, a birefringent polymer is cut into the same shape as the part whose stresses are of interest. The model is then loaded in a loading frame and the whole setup is placed in a polariscope (see figure below) where a beam of polarized light is directed through it onto a screen and is photographed. The fringes appearing in the photgraph which are produced as a result of the difference in speed and polarization of the two rays are related to the difference in principal stresses at various points of the model.